3.3 Completed Notes

3.3: Multiplication and Division of Whole Numbers

Definition: (Multiplication of Whole Numbers) For any whole numbers a and n, where $n \neq 0$, $n \times a = a + a + ... + a$ (n terms). This can also be written $n \cdot a$ or just na. for $n=0$, $0 \times a = 0$

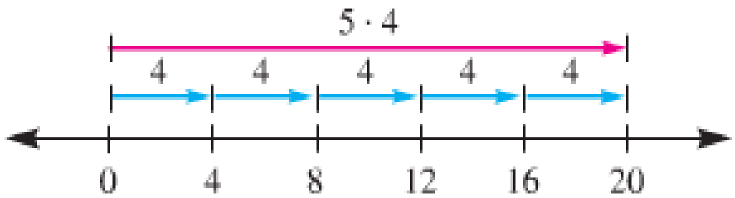
Definition: The numbers n and a are called the factors and na is called the product.

Multiplication Model 1: Representing the product of two numbers by adding numbers multiple times is known as the repeated addition model

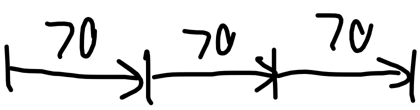
Example: A king size candy bar costs \$1. If you have a huge craving for chocolate and buy five of these, how much do you spend?

\$1 + \$1 + \$1 + \$1 + \$1

Multiplication Model 2: The number line model is another way to represent multiplication, in which we draw a number line and represent numbers by arrows pointing right whose length is the same as the second number. We place arrows for the second number one after the other, and there are as many arrows as the first number. The product is the whole length.



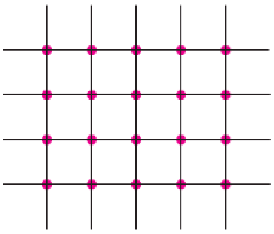
Example: If you drive down the interstate at 70 mph for 3 hours, how far did you go?

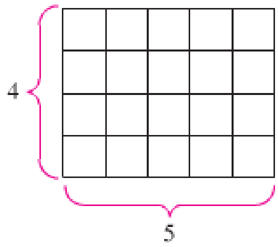


Note: The repeated addition and number line models are virtually the same thought process, so either answer is acceptable to me when one works.

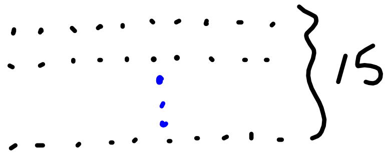
3.3 Completed Notes

Multiplication Models 3 and 4: The array model and the area model are used to represent when a situation is easily arranged into a grid with rows and columns.

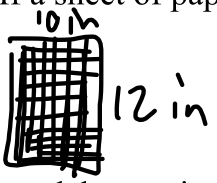
Array: 

Area: 

Example: (Array Model) If you have a sheet of stickers with 15 rows of 10 stickers, how many stickers are on the sheet?



Example: (Area Model) If a sheet of paper is 10 in. by 12 in., what is the area of the sheet of paper?



Note: The array and area models are virtually the same thought process, so either answer is acceptable to me when one works.

Multiplication Model 5: The Cartesian Product Model is used to represent when you have two sets and you are looking at all combinations of elements where 1 element is from the first set and 1 element is from the second set.

Example: Tyler has 20 shirts and 8 pairs of pants. How many ways can he combine them for one outfit?

{red shirt, green shirt, ...} (18 more)

{jeans, khakis, ...} (6 more)

Combinations: (red shirt, jeans), (green shirt, jeans), (red shirt, khakis)

⋮

$20 \times 8 = \boxed{160}$

Example: Determine which models you would use for each situation.

(a) Your room is 12 ft by 14 ft. What is the area of your floor?

area

(b) Water is flowing out of the faucet at 300 mL per second for 30 seconds. How much water comes out of the faucet?

number line

(c) A refrigerator magnet weighs 3 ounces. If you have 4 of these, how much do they weigh altogether?

repeated addition

(d) You have 10 different pairs of socks that go well with any shoe and you have 1,394 pairs of shoes. How many ways can you fashionably cover your feet?

cartesian product

(e) A base ten flat is actually composed of units. Each row has 10 units and each column has 10 units. How many units are in a flat?

array

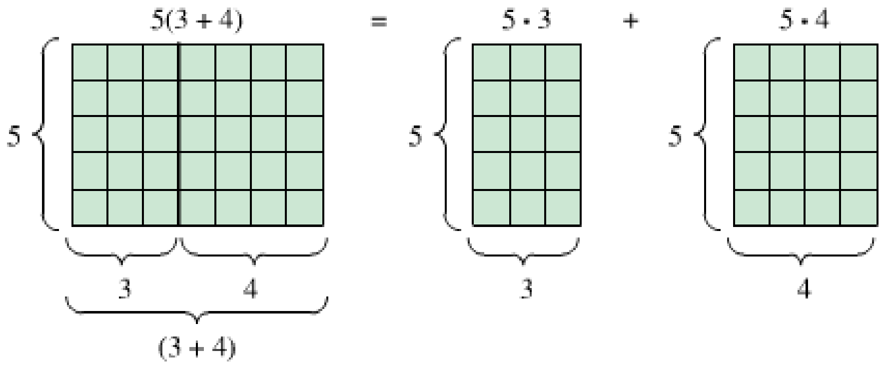
Theorem: The following properties hold for multiplication of whole numbers:

1. (Closure) If a and b are whole numbers, then $a \times b$ is a whole number.
2. (Commutative) If a and b are whole numbers, then $a \times b = b \times a$.
3. (Associative) If a, b, and c are whole numbers, then $(a \times b) \times c = a \times (b \times c)$.
4. (Identity) There is a unique whole number (1 in this case), the multiplicative identity, such that for any whole number a, $a \times 1 = 1 \times a = a$.
5. (Zero Product) For any whole number a, $a \times 0 = 0 \times a = 0$.

Theorem: The following properties hold for multiplication of whole numbers:

6. (Distributive) For any whole numbers a, b, and c, $a(b + c) = ab + ac$. Similarly, $a(b - c) = ab - ac$.

Rationale:



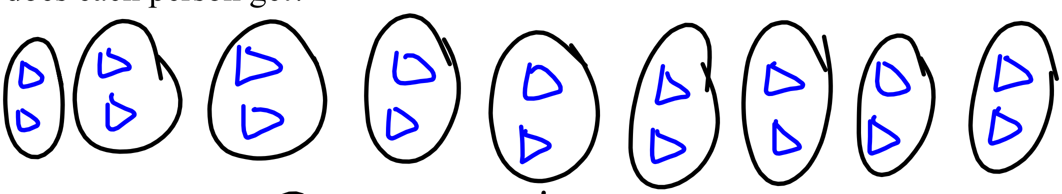
Definition: (Division of Whole Numbers) For any whole numbers a and b, where $b \neq 0$, $a \div b = c$ if and only if c is the unique whole number such that $b \times c = a$.

Definition: The number a is called the dividend, the number b is called the divisor, and the number c is called the quotient.

Division Model 1: The partition model is used to represent when you have a set of elements and wish to distribute them equally (partition) them into smaller groups. You then determine how many elements are in each group.

given number of groups

Example: You baked a birthday cake and cut it into 18 pieces. If there are 9 people eating birthday cake and they eat equal portions, how many pieces does each person get?

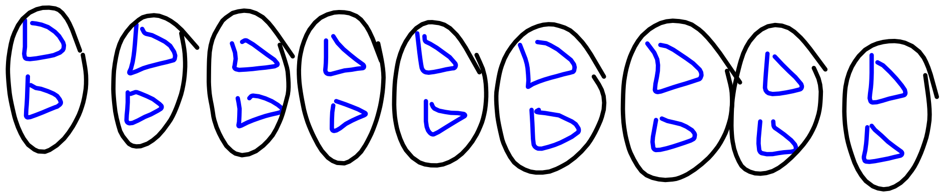


2 in each group

Division Model 2: The measurement model is used to represent when you have a set of elements and wish to make groups with the same amount in each group. You then determine how many groups you can make.

given amount in each group

Example: You baked a birthday cake and cut it into 18 pieces. If you give 2 pieces to each person, how many people can you serve?



9 groups

Example: Determine which models you would use for each situation.

A group of 10 people run a car wash and make a $620 profit. If they split the profits equally, how much should each person get?

partition: 10 groups, put $62 in each group

You have a bucket containing 150 pieces of Halloween candy. If you give 5 pieces of candy to each trick-or-treater, how many trick-or-treaters can you give candy to?

measurement: 5 pieces in each group, makes 30 groups

Division with Zero: Let n be a whole number.

$0 \div n = 0$ (if $n \neq 0$) because the only c that satisfies $nc = 0$ is $c = 0$.

How about $n \div 0$? Why is it undefined?

$n \neq 0$: Want c such that $n \div 0 = c$.

So, $0 \times c = n$. There is no c to do this

$n = 0$: Want c such that $0 \div 0 = c$.

So, $0 \times c = 0$. However, this c is not unique.